Total No. of Questions - 12]   [Total No. of Printed Pages-2

B.A./B.Sc. VI Semester Examination

CBS-VI/6(R&P)

214922

MATHEMATICS

Course No. : UMTTE - 602

Time Allowed- 2½ Hours   *Maximum Marks- 80*

*Note: Attempt **all** questions from Section A and B and any **two** questions from Section C.*

SECTION - A

Note: Attempt **all** questions from this section. Each question carries **04** marks. **(5×4=20)**

1. Show that $\left[\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right]^8=\cos 8\theta-i\sin 8\theta$.

2. If cosh$x$=sec y, show that $\tan h^2\frac{x}{2}=\tan^2\frac{y}{2}$.

3. Show that $f(z)=e^z$ satisfies Cauchy - Riemann equations. Also find $f'(z)$.

4. Using definition of the integral of f(z) on a given path, evaluate $\int_{-2+i}^{5+3i} z^3dz$.

5. Find the radius of convergence of the series $\sum_{n=2}^{\infty}\frac{z^n}{\log n}$.

3000   [Turn Over

SECTION - B

Note: Attempt **all** questions from this section. Each question carries **08** marks. **(3×8=24)**

6. Prove that

$$\sin^9\theta=\frac{1}{256}[\sin 9\theta-9\sin 7\theta+36\sin 5\theta-84\sin 3\theta+126\sin\theta].$$

7. If $\tan(x+iy)=\sin(u+iv)$, prove that $\frac{\sin 2x}{\sin h2y}=\frac{\tan u}{\tan hv}$.

8. State and prove minimum modulus principle.

SECTION - C

Note: Attempt any **two** questions from this Section. Each question carries **18** marks. **(2×18=36)**

9. a) Prove that the roots of the eq $(5+x)^5=(5-x)^5$ are given by $x=5i\tan\frac{r\pi}{5}$, r = 0, 1, 2, 3, 4.

   b) Show that

   $2^5\sin^4\theta\cos^2\theta=\cos 6\theta-2\cos 4\theta-\cos 2\theta+2$. **(9,9)**

10. a) If $i^{\alpha+i\beta}=\alpha+i\beta$, prove that $\alpha^2+\beta^2=e^{-(4n+1)\beta\pi}$.

    b) If $\tan(\theta+i\phi)=\sin(x+iy)$, prove that $\cot hy.\sin h2\phi=\cot x.\sin 2\theta$. **(9,9)**

11. State and prove Cauchy - Goursat theorem.

12. a) State and prove Liouville's theorem.

    b) Show that $f(z)=|z|^2$ is continuous $\forall z$ but nowhere differentiable except at z = 0. **(10,8)**