B.A. / B.Sc. VI Semester Examination

BS-VI/7(B)

222247

MATHEMATICS

Course No. : 601

Time Allowed: 3 hours | Maximum Marks: 80

Note: Do any five questions, selecting one from each unit.

Unit – 1

1. a) Let V be a vector space over a field F, then show that

   i) x + y = x + z → y = z, ∀ x, y, z ϵ V.

   ii) a0=0, ∀ a ϵ F, where 0ϵV.

   iii) a(-x)=-(ax), ∀ a ϵ F and xϵV.

   iv) ax=0→ either a=0 or x=0, where a ϵf and x ϵV.

   b) Prove that if U and W are subspaces of a vector space V (F), then U w and U ∩ W are also subspaces of (F).

2. a) Define the linear span L(S) of a non-empty subset S of a vector space V(F). Show that is a subspace of V(F) if and only if L(S)=S. https://www.jktopper.com

   b) Prove that the vectors (1,2, -3), (1.-3,2), (2,-1,5) in V_3(R) are linearly independent.

Unit – 11

3. a) Suppose a basis of vector space (F) contains n elements. Then prove that.

   i) A subset T of V having more than a elements is linearly dependent

   ii) A linearly independent subset T of V cannot have more than elements.

   b) Prove that the set S={1.x.x^2.....x^n) of (n+1) polynomials is a basis for the vector space P_n(R) of all polynomials of degree at most n over the field R of real numbers.

4. Prove that every n-dimensional vector space over a field F is isomorphic to F.

Unit - III

5. a) Let V (F) and W (F) be vector spaces. Then prove that T:V → W is a linear transformation if and only if T(a v + B w)= a T (v)+BT(v). ∀ v, w ϵ V and a.BϵF.

b) Let V(F) be a vector space and T. S be a linear operators on V. Then prove that if is invertible and 0≠aϵf. then aT is invertible and (a T)' =a 'T.

6. a) If a linear transformation $T:R^3 \rightarrow R$ is defined by T (1,1, 1)-3: T (1,1,0)= -4T (1,0,0)-2, then find the transform of (x 2) under T.

b) Let $T:R^3 \rightarrow R$ defined by T(x,y,z) = (x +3y, 2z – y X+z) be a linear operator. Find the matrix of T w,r,t the basis B = {(1,1,1), (1,1,0) (1,0, 1))

Unit - IV

7. a) Define a denumerable and a non-denumerable set. Prove that Z. is denumerable

b) Define A, for a subset of R. Prove that is open und only if A = A.

8. a) What do you mean by limit point of a set? What is the limit point of a finite set? Find the derived set of N. https://www.jktopper.com

b) Prove that if a subset of R satisfies Heine-Borel property. then it is compact.

Unit - V

9. a) Let X be a non-empty set. Show that a mapping

$d : X \times X \rightarrow \mathbb{R}$ defined by $d(x,y) = \begin{cases} 0, & if\ x = y \\ 1, & if\ x \neq y \end{cases}$ is a

Metric on X.

b) Prove that every closed sphere in a metric space is a closed set.

10. a) Let (X, d_1) and (Y, d_2) be two metric spaces and let ʃ:X→ Y be a mapping. Then prove that ʃ is continuous if and only if the inverse image of an open set in Y under ʃ is an open set in X.

b) Define the interior, closure and boundary of a set in a metric space. Prove that for a subset A in a metric space (X, d),

i) $(\bar{A'}) = (A^0)'$ and

ii) $b(A) = \bar{A} \cap \bar{A'}$, where $b(A)$ is the set of boundary points of A.