Total No. of Questions - 12] [Total No. of Printed Pages-3

B.Sc. IV Semester Examination

CBS-IVs/7(R&P)

222592

MATHEMATICS

Course No. : UMTTC - 401

Time Allowed- 2½ Hours *Maximum Marks- 80*

Note: 1. *Question paper consists of 3 sections, viz, section A, section B and section C.*

2. *Section A consists of 5 questions of 4 marks each. All questions in section A are compulsory.*

3. *Section B consists of 3 questions of 8 marks each. All questions in section B are compulsory.*

4. *Section C consists of 4 questions of 18 marks each. A student is required to attempt any Two questions in section C.*

SECTION - A

(Each question in Section A carries 4 marks. All questions are compulsory).

1. Define order of an element of a group. Find order of $2 \in \mathbb{Z}_6$, where $\mathbb{Z}_6$ is additive group of integers under addition modulo 6.

2. Does there exist a non - abelian cyclic group? Justify.

3. Define Euler's function ϕ. Find $\phi(8)$ and $\phi(5)$.

4. Let $f: G \to G'$ be a homomorphism from group G into G'. Then show that $f(e) = e'$, where e and e' are respective identity elements of G and G'.

5. Show that every ideal of a ring R is a subring of R. Also given an example of a subring of a ring which is not an ideal.

SECTION - B

Each question in Section B carries 8 marks. All questions are compulsory.

6. Prove that any finite semi group is a group if and only if both the cancellation laws hold in it.

7. Prove that intersection of two subgroups of a group is again a subgroup. What about union? Justify.

8. State and prove Fermat's Theorem.

SECTION - C

(Each question in Section C carries 18 marks. Attempt any two).

9. Let G be a group with identify e. Then show that

   i. $a.b = a.c$ implies b = c, for all $a,b,c \in G$.

   ii. $(a.b)^{-1} = b^{-1}.a^{-1}$, for all $a,b \in G$.

   iii. Identity of G is unique.

   iv. $a^{-2} = (a^2)^{-1}$ for all $a \in G$.

10. i. Show that the order of a cyclic group is equal to the order of its generator.

ii. Define commutator subgroup. Show that G is abelian if and only if commutator subgroup of G is trivial.

11. i. Define a quotient group. Give an example of a non-abelian and non - cyclic group G and normal subgroup H of G such that G\H is cyclic.

ii. If H is subgroup of a group G then show that $aH=bH$ if and only if $a^{-1}b \in H$.

iii. If H is subgroup of a group G, then there is one to one correspondence between any two right cosets of H in G.

12. i. Define a ring, an integral domain and a zero divisors in a ring. Show that a commutative ring R with unity is an integral domain if and only if cancellation laws hold in it.

ii. Define a field and show that every finite integral domain is a field.